Measurement of the Electrical Conductivity of Xenon in a Shock Tube

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The electrical conductivity of xenon was measured in a shock tube at gas pressures of about one atmosphere and in the temperature range of 5500°K–9000°K. Where ionization equilibrium was achieved immediately behind the shock front, the experimental conductivities were in agreement with the theoretical analyses based on either the Boltzman equation or the Fokker-Plank equation. At Mach numbers below 8 ionization equilibrium was not always achieved and considerable uncertainty exists in correlating the measured conductivity to known gas properties.

I. Introduction

RECENTLY Devoto[1] reported the results of numerical calculations on the electrical conductivity of krypton and xenon. He used higher approximations to the Chapman-Enskog-Burnett theory than previously available. The results were compared to the shock tube experiments of Johnsen and Rehder[2] in krypton. These experiments were performed in the gas temperature regime where Coulomb collisions control the magnitude of the electrical conductivity. It was found[1] that the experimental conductivities were considerably higher than the theoretical values obtained by Devoto or by the Spitzer and Harm theory.[3] The latter theory applies to a fully ionized gas. Devoto did not compare his results with experimental conductivity measurements in xenon[4] which were published at the same time as Johnsen and Rehder's work.[2] In the previous work,[4] it was reported that for shock Mach numbers M_s greater than 8.5 ionization equilibrium was attained immediately behind the shock front. With the addition of less than 1% of diatomic seed gas, equilibrium behind the shock was attained at Mach numbers as low as 8. The Mach number is defined in the usual way, namely, shock velocity divided by the speed of sound of the stationary gas in front of the shock. Below Mach 8, the electron density and conductivity increased continuously from the shock front to the contact surface. In all cases, the maximum conductivity in a given run was shown. It was concluded that the conductivity of xenon was in agreement with the theoretical analyses of Shkarofsky[5] and with the elastic electron-xenon atom collision cross section data of Frost and Phelps.[6] This paper presents more extensive results on the electrical conductivity in xenon than given previously.[4] Specifically, the accuracy of the shock velocity measurements, with which the conductivity is correlated, was improved; a second method of measuring the shock velocity was used; and the continuum radiation emitted by the plasma was measured in each experiment. In addition, the experimental results are compared with Devoto's theoretical results and with the results of other shock tube studies on the conductivity of noble gases.

II. Experimental Results

The 5 cm × 5 cm square shock tube described in the previous paper[4] was used for the present study. The conductivity was measured as in the previous study by the magnetic-induction, Lin method[7] and by a high-frequency induction technique.[8] The latter method is useful in the conductivity range of 1 to 100 mho/m. The Lin method is most convenient for conductivities greater than 100 mhos/m. The shock velocity was measured with pressure transducers flush mounted along the shock tube wall. At higher Mach numbers, a streak camera was used to deduce the shock velocity from the luminous front in the streak photograph. The latter method was used by Johnsen and Rehder[2] to measure the shock velocity. The experiments were performed at Mach numbers ranging from 7–11.5 in the test section, corresponding to gas temperatures of 5500°K–9000°K. The initial pressure in the driven tube was varied from 2–8 torr. The highest shock Mach number which could be obtained in the shock tube for a reasonable hot gas slug length was 11.5. For M_s less than 9, 0.1% hydrogen seed was added to the xenon to achieve rapid ionization equilibrium behind the shock. The low H_2 concentration has a negligible effect on the plasma properties and it is fully dissociated at gas temperatures above 5000°K.

The following results were obtained:

1) It was found that the measurements of the shock velocity obtained from the streak photographs were on the average 15% below the values obtained with the pressure transducers. The discrepancy can be attributed to the fact that the streak camera records the ionization front velocity and not the shock velocity. Because of the finite ionization relaxation time, the ionization front will be behind the shock front and in the presence of shock attenuation the ionization front velocity will generally differ from the shock velocity.

2) As noted previously,[4] the measured shock Mach number attenuation near the test section was on the average 4% per meter. Part of this attenuation was caused by misalignment of the five foot long, plastic test section with the upstream steel section of the driven tube. Shock attenuation causes the gas properties to vary as a function of distance behind the shock front. Thus to correlate the conductivity data obtained at Mach numbers less than 8, where ionization equilibrium was not achieved immediately behind the shock front, the plasma properties must be known as a function of distance behind the shock. To determine the variation of the plasma properties, the continuum radiation emitted by the shock heated gas at 4777 Å was measured as a function of distance behind the shock front. The details of the measuring technique are given elsewhere.[9] This radiation is theoretically proportional[10] to $N_e^2 T_e^{-0.5}$, where N_e is the electron density and T_e the electron temperature. This relationship of the radiation to the electron density was approximately verified from continuum radiation measurements in xenon at various shock Mach numbers.[9] Assuming

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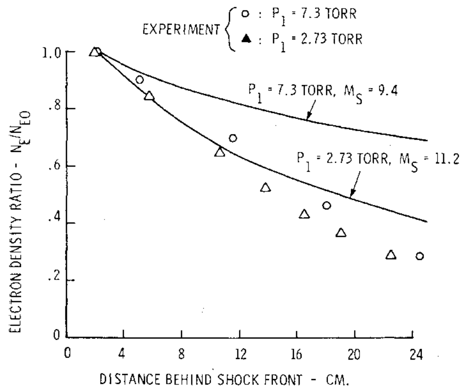

Fig. 1 Experimental and theoretical relative electron density change, obtained from the continuum radiation, as a function of distance behind the shock front. The theoretical curves were obtained with the analyses of Ref. 10 and by using the semiempirical expression, $Q = 2 \times 10^{-35} N_e^2 T_e^{-0.5}$ (joules m^{-3} sec^{-1}), for the continuum radiation loss.

this N_e^2 relationship of the continuum radiation, Fig. 1 was plotted. It shows the ratio of the local electron density N_e divided by the peak electron density behind the shock N_{eo}, as a function of the distance behind the shock front for two experiments. In one case, the initial driven gas pressure p_1 was 2.73 torr and M_s was 11.2. In the other case p_1 was 7.3 torr and M_s was 9.4. The ratio of N_e/N_{eo} is equal to the square root of the ratio of the local to the maximum continuum radiation. Assuming that thermodynamic equilibrium is achieved immediately behind the shock front, one obtains from the shock conservation equations a value of N_{eo} of 2.76×10^{16} cm^{-3} for the 2.73 torr experiment. From Fig. 1, one finds that N_e at the end of the test time in this experiment is equal to $0.31 \times N_{eo}$, or 8.6×10^{15} cm^{-3}. The theoretical conductivities corresponding to these two N_e values (assuming thermodynamic equilibrium) are 2200 and 1330 mho/m, respectively. The computations were done using the Shkarofsky method.[4] The corresponding measured conductivities were 1500 mho/m at the shock front and 900 mho/m at the contact surface. Both these conductivity values are about two-thirds of the theoretical values. Similarly, for the other experiment in Fig. 1 one finds that the theoretical N_{eo} is 1.43×10^{16} cm^{-3}. At the end of the test time, N_e, deduced from the radiation data, is 3.7×10^{15} cm^{-3}. The corresponding theoretical conductivities are 1400 and 780 mho/m, respectively. The measured conductivities were 1200 and 580 mho/m, respectively, which is 86% and 75% of the theoretical values. Possible causes for the somewhat poorer agreement in the $M_s = 11.2$ experiment are discussed in Sec. III. Nevertheless, it is felt that the close agreement between the measured conductivity data and the conductivities deduced from the radiation data, provide further verification that the radiation is proportional to N_e^2.

Knowing the continuum radiation emitted by the plasma, one can compute[10] the radiation cooling effect on the gas properties downstream of the shock front. The continuum radiation power loss Q is given by[10]:

$$Q(\text{joules}, M^{-3}, \text{sec}^{-1}) = 8 \times 10^{-36} N_e^2 T_e^{-0.5} \qquad (1)$$

The theoretical loss given by Eq. 1 was too low to account for the observed N_e decrease. It was found that by increasing the constant in Eq. (1) by a factor of 2.5, reasonably good agreement was obtained in the 2.73 torr experiment, as shown in Fig. 1. This variation in Q is within the range of uncertainty in the theory. However, the same constant, 2×10^{-35}, in the radiation equation gives very poor agreement with the 7.3 torr experiment (see Fig. 1). It was observed that the variation of N_e behind the shock front was a function of the Mach number. Specifically, the deviation of the experimental N_e variation from the values predicted with Eq. (1) increased with decreasing Mach number. The deviation of the experimental N_e variation from theory[10] can be partly attributed to the effect of shock attenuation which was not considered in the theoretical analyses.

In summary, the radiation data showed the N_e^2 dependance predicted by the theory.[10] However, the variation of the radiation with distance behind the shock was an unknown function of Mach number. Therefore, it was not possible to establish the variation of the gas properties with distance behind the shock front. Consequently, the correlation of the conductivity data to specific gas properties in the experiments where the maximum conductivity occurred at the contact surface is subject to considerable uncertainty.

3) Figure 2 shows the experimental and theoretical conductivity of xenon as a function of incident Mach number. The theoretical curves were computed for an initial pressure of 8 torr in front of the shock. The gas temperatures shown on the abscissa correspond to the values obtained at various shock strengths at an initial pressure of 8 torr. For 8 torr, the pressure behind the incident shock increases from 0.55 atm at $M_s = 6.5$ to 2.1 atm at $M_s = 12$. Devoto's theoretical conductivity data[1] which is given for 1 atm, was corrected for the preceding pressure variation as follows: above Mach 7.5, the effect on the conductivity of the deviation of the pressure from one atm. is negligible. Below Mach 7.5, Devoto's data was multiplied by the inverse of the square root of the pressure ratio to account for the pressure effect on the electron density and the electron-neutral collision frequency. The triangular experimental points in Fig. 2 apply to experiments at initial gas pressures of 7 to 8 torr and the circular points to initial pressures of 2.5–5.5 torr, The theoretical conductivities at the lower initial pressures are up to 8% below the 8 torr theoretical curves for Mach numbers 9.5–12 and up to 10% above the theoretical curves for Mach numbers between 8 and 9.5. The solid points in Fig. 2 correspond to experiments in which equilibrium condi-

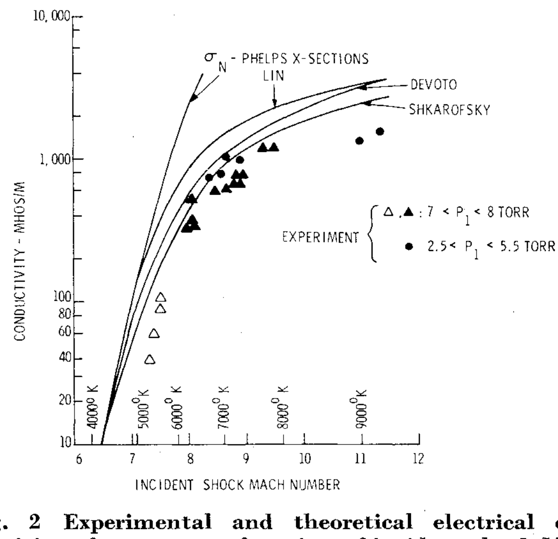

Fig. 2 Experimental and theoretical electrical conductivity of xenon as a function of incident shock Mach number. σ_N is the electron-neutral xenon conductivity obtained with Frost and Phelps' cross sections.[6] The three other theoretical curves were obtained at a p_1 of 8 torr using the same neutral cross sections[6] and the methods of Lin,[7] Devoto,[1] and Shkarofsky[5] to obtain the total conductivity. Note that the two data points at $M_s > 11$ represent the results of over a dozen experimental runs. The solid points represent experiments where equilibrium ionization was reached immediately behind the shock. For the open points ionization equilibrium was not achieved during the test time.

tions were reached immediately behind the shock. These data points represent the maximum conductivity, which occurred immediately behind the shock front. The two data points for $M_s \geq 11$ are representative of over a dozen experimental runs. The maximum conductivity for $M_s \geq 11$ was within 10% of 1500 mho/m. As already noted, no data was obtained for $M_s \geq 11.5$ because of shock tube limitations. In addition to possible errors in the calibration of the Lin probes, the major cause of uncertainty in the data points above Mach 8 is due to shock attenuation. In all the experiments, the shock Mach number in the 50-cm region upstream and downstream of the conductivity probe was used. Above Mach 8 the estimated error in the abscissa is ±2%. Below Mach 8 the conductivity increased continuously toward the contact surface. As discussed in Sec. IIa the error in assigning a Mach number to these data points is considerably greater. These low Mach number results should therefore be viewed with caution. The same comments apply to the previously give data.[4]

III. Comparison with Theory and Other Experiments

Four theoretical computations were used for comparison with the experiments. In all four cases, the electron-xenon elastic collision cross sections given by Frost and Phelps[6] were used. The Shkarofsky method[5] uses the Fokker-Plank equation to derive the conductivity. The advantage of the Shkarofsky method is its simplicity. One computes the Maxwell averaged electron-neutral, $\langle \nu_{en} \rangle$, and electron-ion collision frequencies, $\langle \nu_{ei} \rangle$. A single exponential fit must be found for the velocity dependent electron-neutral collision frequency, ν_{en}, in the energy range of interest. In Ref. 4, this exponent is given by the expression: $\nu_{en} = Kv_e^{4.46}$, for the electron energy range of 0.7–2 ev. K is a constant and v_e is the electron velocity. Using the least mean square method, it was found that $\nu_{en} = Kv^{3.7}$ can be used to fit (within 30%) the electron-xenon cross section[6] from 0.5–8 ev. Shkarofsky tabulates the functions necessary to compute the conductivity as a function of the exponent of the electron velocity in the range v_e^{-3}–v_e^3. For exponents greater or equal to 3, the method does not converge well in the limit when the effect of Coulomb collisions goes to zero. In the range of the present experiments, the ratio of $\langle \nu_{en} \rangle / \langle \nu_{ei} \rangle$ ranged from 50 at $M_s = 6.5$ to 0.1 at $M_s = 11.5$. At a collision frequency ratio of 50, the effect of Coulomb collisions is still of sufficient magnitude to obtain convergence. In computing the Shkarofsky curve in Fig. 2, the tabulated values[5] for v_e^3 were used. Full details of the computations are given elsewhere.[4] The second theoretical computation shown in Fig. 2 is the electron-atom collision dominated conductivity, σ_N. σ_N is derived by Allis[11] from a spherical harmonic expansion of the Boltzmann equation, neglecting electron-electron interactions. σ_N was computed numerically by assuming a Maxwellian distribution. Using this value of σ_N, the effect of Coulomb collisions was included by using the Lin method[7] in which the resistivities due to electron-atom collisions and electron-ion collision are added. The latter resistivity was computed from the Spitzer and Harm theory.[3] The fourth theoretical curve was obtained from Devoto's[1] tabulated values for the conductivity of xenon at one atm. The results were adjusted to the pressure behind a shock propagating into an 8 torr stationary gas, as outlined in Sec. II. One notes in Fig. 2 that at low conductivities where electron-atom collisions dominate the Devoto curve and the σ_N curve converge since the two computations are similar. The Shkarofsky curve intersects the other curves at 10 to 20 mho/m. At lower conductivities the Shkarofsky method would overestimate the conductivity due to the convergence difficulties mentioned previously.

In the Coulomb collision dominated region, the Shkarofsky and Lin (i.e., Spitzer and Harm) curves should converge since the two computations are similar.[5] However, at Mach 11.5, where $(\langle \nu_{en} \rangle / \langle \nu_{ei} \rangle) = 0.1$, the effect of electron-neutral collisions is of sufficient magnitude to depress the Shkarofsky curve below the Lin curve. At M_s of 11, the Devoto curve intersects the Lin curve. At higher conductivities, Devoto's method gives higher conductivities than the Spitzer and Harm method.[1] It can be seen from Fig. 2 that the summation method of Lin overestimates the conductivity in the region where electron-atom and electron-ion collisions are of equal magnitude, i.e., $7 < M_s < 10$. Figure 2 shows that in the region where equilibrium ionization was achieved immediately behind the shock, i.e., for $M_s > 8$, the Shkarofsky method gives the best agreement with the experiments. The data points at $M_s > 11$ lie below the theoretical curve by about 33%. A possible cause for this discrepancy could be the strong radiation cooling occurring in these high electron density experiments. The axial resolving power of the Lin conductivity probe was between 2.5 and 3 cm. As can be seen in Fig. 1 in the $M_s = 11.2$ experiment, the maximum electron density decreases by about 15% to 20% in 3 cm. This rapid change in n_e could possibly influence the conductivity probe response. In addition, the ionization processes occurring at the high Mach numbers are very fast. Thus, it is not certain that the electron density reaches the theoretical value corresponding to the measured Mach number before the radiation cooling effect begins to dominate the magnitude of the electron density. Whatever the cause of the discrepancy, it is to be emphasized that in dozens of runs at these high Mach numbers the measured conductivities were always below the values predicted by Shkarofsky.

The present experimental results are consistent with the experimental results obtained by Lau[12] and Lin[7] in argon. Their results agree reasonably well with the Shkarofsky method in those cases where equilibrium ionization occurred during the test time. The experimental results of Johnsen and Rehder[2] in krypton and Pain and Smy[13] in argon in the Coulomb collision dominated, conductivity regimes were considerably above theoretical.[1,14] In both these experiments, streak cameras or photomultipliers were used to measure the shock velocity. As noted in Section II.1 it was found that the optical shock velocity measurements were about 15% below the values obtained with pressure transducers. A 15% increase in the Mach number would bring their data[2,13] in reasonably good agreement with the theories.[1,3,5] In connection with the work of Johnsen and Rehder,[2] the authors observe that the rate of the measured conductivity decrease behind the shock is much faster than the value deduced from the continuum radiation data. In the present study the two measurements agree (Section II.2). They attribute this large conductivity decrease to boundary-layer growth behind the shock, which influences the signal of the Lin probe. However, using the computation of Duff[15] one obtains a boundary-layer thickness which is one-third of the value given in Ref. 2. Thus the boundary layer effect on the probe signal is considerably less than they assumed. In a recent study[16] in krypton at $p_1 = 10$ torr and $M_s = 11.6$, Rehder, et al. report that the constant in the radiation equation, which for krypton is 9.29×10^{-36} (see Eq. 1), must be multiplied by two to obtain agreement with the measured radiation data. Applying the same correction to radiation data given in Ref. 2 for krypton at $p_1 = 5$ torr and $M_s = 13.3$, the present author finds a discrepancy of about 25% between the computed and the experimental radiation data. This dependance of the radiation cooling the Mach number and initial gas pressure is in agreement with the present results as shown in Fig. 1. It verifies one of the conclusions of this study, namely, that conductivity data obtained far downstream of the shock front are subject to considerable uncertainty.

IV. Conclusions

The results of this study and the other shock tube studies on the electrical conductivity in the noble gases are in general agreement with the theoretical computations of Spitzer and Harm,[3] Devoto[1,14] and Shkarofsky.[5] The present author prefers the Shkarofsky method because it is simple to use. It has been shown that where a large discrepancy with the preceding theories was found[2,13] one can attribute it to uncertainties in the shock velocity measurement. The present study also showed that in those cases where the ionization level increased continuously toward the contact surface considerable uncertainty existed in correlating the measured conductivity to known gas properties. Finally, it was observed that the radiation cooling in the shock heated gas slug was a function of initial pressure and Mach number.

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